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MULTI-FUZZY SUBGROUP AND ITS PROPERTIES

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Abstract

In this paper we introduce the concept of multi- fuzzy subgroup and their some related properties are investigated and we also define the algebraic structures of multi- fuzzy subgroup. The purpose of this study is to implement the fuzzy set theory and group theory in multi fuzzy groups

Research Objectives : Fuzzy mathematics is a branch of mathematics that is related to fuzzy logic. It started in 1965 after the publication of Lotfi Asker Zadeh's seminal work Fuzzy set.

Before knowing Fuzzy group we should know the Fuzzy set. Fuzzy group is an extension of theories of fuzzy sets. In this paper we define a new algebraic structureof multi-fuzzysubgroup and multi-anti-fuzzy subgroups and study some of theirrelated properties.

Hypothesis : A hypothesis is an unproven statement which is supported by all the available data and by much weaker result. It is a sequential process of research. It is an indicator of the work to be done for research. It describes the theoretical

Key Words : *Fuzzy set, Multi- Fuzzy set, Fuzzy subgroup, Normal Fuzzy subgroup, Anti Fuzzy Subgroup, Multi-Fuzzy subgroup, Multi Anti Fuzzy subgroup.*

information in relation to how the research study will work. The hypothesis is a formal suggestion that express the outline of the studys proposal. Since Zadeh introduced the concept of a fuzzy set in 1965, various algebraic structures have been fuzzified. As a result the theory of fuzzy group was developed. In 1971 Rosenfield introduced the notion of a fuzzy subgroup and thus initiated study of fuzzy group. In this paper we preset the further investigation into properties of multi-fuzzy subgroup and multi-anti-fuzzy subgroup.

Research Method : A research design is the set of methods and procedures used in collecting and analyzing measures of variables specified in the problem research. A research design is a frame work that has been created to find answers to research questions. There are numerous types of research design that are appropriate for the different types of research projects. The choice of which design to apply depends on the problems posed by the research aim.

Each type of research design has a range of research methods that are commonly used to collect and analyses the type of data, which is generated by the investigations. Here is a list of some of more common research design, which is used to find expected outcome of my proposed work.

1. **Historical** : This aim at systematic and objective evaluation and synthesis of evidence in order to establish facts and draw conclusion about past events. It uses primary historical data, such as archaeological remains as well as documentary sources of the past, it is usually necessary to carry out tests in order to check the authenticity of these sources.

2. **Descriptive** : This design relies on observation as a means of collecting data. It attempts to examine situations in order to establish. Observation can take many forms depending on the type of information sought, people can be interviewed questionnaires distributed.

3. **Correlation** : This design is used to examine a relationship between two concepts. There are two broad classifications of relational statements and association between two concepts.

4. **Comparative** : This design is used to compare past and present or different parallel situations. It can look at situation at different scales, macro (international,

national) or micro (community, individual).

1. Introduction

The concept of fuzzy set was introduced by Lotti A Zadeh in 1965.The theory of multi-fuzzy set is an extension of theories of fuzzy set. And the theory of multi fuzzy sets is proposed by the S. Sabu and T.V. Ramakrishnan.
The investigated the theory of multi-dimensional membership function and some properties of multi- level fuzziness. R. Muthuraj and S. Balamurgan proposed multi-fuzzy group and its level subgroups .The concept of anti-fuzzy subgroup was introduced by BISWAS. In this paper we define a new algebraic structure of multi-fuzzy group and their subgroup and also study some of their related properties.

2. Preliminaries

In this section we explain the some fundamental definitions these are given below.

- **Fuzzy Set** : Let A is a non-empty set and the function $M : A \to [0, 1]$ is called fuzzy set in A.

Remark : If we want to know the difference between the fuzzy set and ordinary set, we observe that when A is a set in ordinary sense of term , so it membership function can take only two values 0 and 1 with a characteristic function then $A(x) = \{0, 1\}$ for all $x \in \{0, 1\}$ for all $x \in G$ while if A is a fuzzy set in G then $0 \leq Ax \leq 1$ for all $x \in G$. Thus the ordinary set becomes a special case of fuzzy set.

- **Multi-fuzzy Set** : Let X be a non-empty set. A multi-fuzzy set A in X is defined as a set of ordered sequence.

$$A = \{(x, \mu_1(x), \mu_2(x) \cdots \mu_i(x) \dots) : x \in X\}$$

where $\mu_i : X \to [0, 1]$ for all i.

Remark : (i) If the sequences of the membership functions have only K-terms (finite number of terms) Then K is called the dimension of A.
(ii) The set of all multi fuzzy sets in X of dimension K is denoted by $M^kFS(X)$.

3. Properties of Multi-Fuzzy Sets

Let A and B be any two multi-fuzzy sets of a non-Empty set X then

(i) $A \cup B = A, \quad A \cup A = A$

(ii) $A \subseteq A \cup B, \quad B \subseteq A \cup B, \quad A \cap B \subseteq A$ and $A \cup B \subseteq B$

(iii) $A \subseteq B$ iff $A \cup B = B$

(iv) $A \subseteq B$ iff $A \cap B = A$.

- **Fuzzy subgroup**:

  Let G be a Group and A is a fuzzy subset of G then A is called a fuzzy subgroup of G if

  (i) $A(x, y) \geq \{A(x), A(y)\}$

  (ii) $A(x^{-1}) = A(x)$.

- **Normal fuzzy subgroup** : Let G be a Group and A is fuzzy subgroup of G, so A is also satisfied the above fuzzy subgroup conditions then A is called normal fuzzy subgroup of G if $A(x, y) = A(yx)$ for all x and y in G.

- **Anti-fuzzy subgroup** : Let A be a fuzzy set on a group G, then A is called an anti-fuzzy subgroup of G, if for all $x, y \in G$.

  (i) $A(xy) \leq max\{A(x), A(y)\}$

  ((ii) $A(x^{-1}) = A(x)$

- **Multi-fuzzy subgroup** : A multi fuzzy set A of a group G is called a Multi-fuzzy subgroup of G if for all $x, y \in G$,

  (i) $A(xy) \geq \min\{A(x), A(y)\}$

  (ii) $A(x^{-1}) = A(x)$.

- **Multi-anti-fuzzy subgroup** : A multi- anti-fuzzy set A of a group G is called multi-anti-fuzzy subgroup of G if for all $x, y \in G$.

  (i) $A(xy) \leq \max\{A(x), A(y)\}$.

(ii) $A(x^{-1}) = A(x)$.

4. Properties of Multi-fuzzy and Multi-anti-fuzzy Subgroups

In this section, we discuss the some properties of Multi-fuzzy and multi-anti fuzzy subgroups.

Theorem : Let A be a multi-fuzzy subgroup of a group G and "e'' is the identity element of G. Then

(i) $A(x) \leq A(e)$ for all $x \in G$.

Proof : Let $x \in G$.

$$\begin{aligned} A(x) &= \min\{A(x), A(x)\} \\ &= \min\{A(x), A(x^{-1})\} \\ &\leq A(x^{-1}) \\ &= A(e). \end{aligned}$$

(ii) The subset $H = \{x \in G/A(x) = A(e)\}$ is a subgroup of G.

Proof : Let $H = \{x \in G/A(x) = A(e)\}$.

Clearly H is non-empty as $e \in H$.

Let $x, y \in H$ then, $A(x) = A(y) = A(e)$

$$\begin{aligned} A(xy^{-1}) &\geq \min\{A(x), A(y^{-1})\} \\ &= \min\{A(x), A(y)\} \\ &= \min\{A(e), A(e)\} \\ &= A(e). \end{aligned}$$

That is, $A(xy^{-1}) \geq A(e)$ and obviously $A(xy^{-1}) \leq A(e)$ BY (i).

Hence, $A(xy^{-1}) = A(e)$ and $xy^{-1} \in H$.

Clearly, H is a subgroup of G.

Theorem : A is a multi-fuzzy subgroup of G iff A^C is a multi-anti fuzzy subgroup of G.

Proof : Suppose A is a multi-fuzzy subgroup of G. Then for all $x, y \in G$.

$$\begin{aligned} A(xy) &\geq \min\{A(x), A(y)\} \\ \Leftrightarrow 1 - A^C(xy) &\geq \min\{(1 - A^C(x)), (1 - A^C(y))\} \\ \Leftrightarrow A^C(xy) &\leq 1 - \min\{(1 - A^C(x)), (1 - A^C(y))\} \\ \Leftrightarrow A^C(xy) &\leq \max\{A^C(x), A^C(y)\}. \end{aligned}$$

We have, $A(x) = A(x^{-1})$ for all x in G.

$$\Leftrightarrow 1 - A^C(x) = 1 - A^C(x^{-1}).$$

Therefore $A^C(x) = A^C(x^{-1})$.

Hence A^C is a multi-anti fuzzy subgroup of G.

Theorem : Let A be any multi-anti fuzzy subgroup of a group G with identity "e''. Then $A(xy^{-1}) = A(e) \Rightarrow .A(x) = A(y)$ for all x, y in G.

Proof : Given A is a multi-anti fuzzy subgroup of G and $A(xy^{-1}) = A(e)$. Then for all x, y in G

$$\begin{aligned} A(x) &= A(x(y^{-1}y)) \\ &= A((xy^{-1})) \\ &\leq \max\{A((xy^{-1}), A(y))\} \\ &= \max\{A(e), A(y)\} \\ &= A(y). \end{aligned}$$

That is $A(x) \leq A(y)$.

Now, $A(y) = A(y^{-1})$, since A is a multi-anti fuzzy subgroup of G.

$$\begin{aligned} &= A(ey^{-1}) \\ &= A((x^{-1}x)y^{-1}) \\ &= A(x^{-1}(xy^{-1})) \\ &\leq \max\{A(x^{-1}), A(xy^{-1})\} \\ &= \max\{A(x), A(e)\} \\ &= A(x). \end{aligned}$$

that is $A(y) \leq A(x)$.
Hence, $A(x) = A(y)$.

Theorem : A is a multi-anti fuzzy subgroup of a group G

$$\Leftrightarrow A(xy^{-1}) \leq \max\{A(x), A(y)\} \text{ for all } x, y \text{ in } G.$$

Proof : Let A be a multi-anti fuzzy subgroup of a group G. Then for all x, y in G,

$$A(xy) \leq \max\{A(x), A(y)\}$$

and $A(x) = A(x^{-1})$.
Now,

$$\begin{aligned} A(xy^{-1}) \leq \max\{A(x), A(y^{-1})\} &= \max\{A(x).A(y)\} \\ \Leftrightarrow A(xy^{-1}) &\leq \max\{A(x), A(y)\}. \end{aligned}$$

5. Conclusion

In this paper we conclude that the concept of multi-fuzzy subgroup, multi-anti fuzzy subgroup and proved their some properties of this new concept. We also give the basic definition of fuzzy set, fuzzy group and their different types. We present a further investigation into properties of multi-fuzzy group and multi-anti fuzzy group. Finally we extend the basic concept of group theory into multi-fuzzy set.

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